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D-Branes and Twelve Dimensions

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Abstract

We study the D-brane solutions to type IIB superstring in ten dimensions and find interpretation in terms of compactification of a twelve dimensional three-brane of (a specific) F-theory on a torus T^2. In this frame-work, there also exist a two-brane which may be argued to be equivalent to the three-brane by utilizing the electric-magnetic duality in eleven dimensions. In this context, we propose for the existence of an isometry in one of the transverse directions to the three-brane in F-theory. As a consequence the two-brane may be identified with the three-brane in twelve dimensions itself. The twelve dimensional picture of D-branes in type IIB theory suggests for the reformulation of type IIB superstring in terms of three-brane of F-theory.

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I. INTRODUCTION

In the recent years, string dualities [1–8] can be understood by assuming the existence of higher dimensional theories. There are several connections between superstring theories in various dimensions [8–12] and it is conjectured that all the string theories are different phases of a single underlying theory in eleven or twelve dimensions. At present, the eleven dimensional quantum theory is know as M-theory in literature [5] whose low energy description is believed to be the eleven dimensional supergravity theory [13]. On the other hand, the twelve dimensional theory known as F-theory [14] is understood as a class of type IIB superstring compactifications containing 7-branes where the dilaton and the Ramond-Ramond (RR) scalar (axion) vary on the internal manifold [14–21]. Thus the 8-dimensional world-volume of 7-branes of type IIB string on a D-manifold S^2 is interpreted as the compactification of F-theory on $K3$ which is in turn dual to heterotic string theory on a two dimensional torus T^2. The duality between F-theory on $K3$ and type IIB on T^2 has been established near the orbifold limit of $K3$ [18]. In this context, we consider a (specific) F-theory in twelve dimensions [14–17] on a torus T^2 which is an artifact underlying the $SL(2,Z)$ invariance [5–8] of the type IIB theory in ten dimensions. We analyze the D-brane solutions to the bosonic sector of type IIB in ten dimensions and interpret their origin in the three-brane solutions to the low-energy limit of a hypothetical F-theory. In order to arrive at a plausible unified picture of three-brane in twelve dimensions itself, we propose for the existence of an isometry in one of the transverse directions to branes.

Among various other recent developments of string theory, the extended solutions namely p-branes with RR charges have played an important role in understanding the non-perturbative effects [22] required by string duality [23]. These objects have been studied considerably and within open string theory known as Dirichlet branes (D-branes). For a review see ref. [24]. It is observed that at low energies below string scale, the dynamics of D-branes is governed by the effective theory of massless modes of the open strings whose end points lie on the $(p+1)$-dimensional worldvolume of the D-(p)brane. In fact the open string state is described by Dirichlet boundary conditions for $(9-p)$-transverse directions and Neumann boundary conditions for $(p+1)$ worldvolume directions. In a simple case of a single p-brane, the effective theory is the reduction of a ten dimensional $N=1$ supersymmetric abelian gauge theory to $(p+1)$ dimensional world-volume of the brane [25]. The interaction between D-branes by means of open string calculations have been studied in the current literatures [25–27]. Interestingly enough, the scattering of closed string states from a quantized D-particle has been discussed recently [28].

In type II theories, RR charges are associated with $(p+1)$ form potentials which are

carried by the D-(p)branes of the theory. These include the p-branes for $p = 0, 2, 4, 6, 8$ in type IIA and $p = -1, 1, 3, 5, 7, 9$ in type IIB theories. In this context, various D-brane and their T-duals have been analyzed [29]. The exact type IIB RR backgrounds have also been addressed using geometrical arguments [30]. It is known that, only D-(p)branes with $0 \leq p \leq 6$ have a natural Minkowski space interpretation as the dual of a p-brane in $D = 10$ dimensions is a $(6 - p)$ brane. Nonetheless, type IIB 7-brane has a (-1)-brane dual which are sources of the RR charge and has an interpretation as D-instanton [31]. Furthermore the dual of type IIA 8-brane [32] is interpreted as a cosmological constant in massive type IIA supergravity and type IIB 9-brane can be viewed as $D = 10$ space-time. Generally, the non-triviality in these $D(p)$-brane solutions is due to the harmonic function of the transverse $(9-p)$ spatial coordinates defining the radial direction which is due to the BPS nature of the solution. It is observed that for $p \leq 6$ the harmonic functions are asymptotically flat and for $p \geq 7$ the asymptotic properties are of different nature. The p-volume tension of D-(p)brane is found to be $\frac{1}{g_s}$ as they are BPS saturated [8]. Thus in the weak coupling limit $(g_s << 1)$ D-branes are localized, however extremely small compared to Nevue-Schwarz Nevue-Schwarz (NS-NS) p-branes and their dynamics have been studied [27]. As a consequence the non-perturbative objects (D-branes) are considered to be intermediate between the fundamental string and solitonic five brane. In addition to D-branes, an understanding of M-branes have been analyzed [33] and a super-matrix formulation of M-theory has been suggested [34]. In this context a matrix model underlying the M-theory [35] and type IIB theory [36] have been proposed.

In the recent past there are various interesting proposals underlying the twelve dimensional origin of type IIB theory [37]. In an interesting paper [38], the D-instanton of type IIB theory is viewed as a wave in twelve dimensions. It is known that the field equations of the type IIB theory [39] can not be obtained from a ten dimensional covariant action. However with vanishing five-form (self-dual) field strength the equations of motion can be derived from an action. The bosonic sector of the type IIB is known to possess an exact global $SL(2, Z)$ invariance [5–7]. Under this ten dimensional $SL(2, Z)$ $S-$duality, the complex scalar (say λ) formed out of an RR scalar (χ) and the dilaton (Φ); $i.e. \lambda = \chi + ie^{-\Phi}$ undergoes a fractional linear transformation and is identified as the modulus of a torus T^2. The two two-form potentials, one in the NS-NS sector and the other in the RR sector get exchanged and the metric along with the four-form potential remain invariant. In order to understand the $SL(2, Z)$ invariance of type IIB in ten dimensions, one compactifies M-theory on a torus to nine dimensions and type IIB on a circle. Then $SL(2, Z)$ of type II theory in nine dimensions gets interpreted as the symmetry of the torus [5,6]. However, only in the limit of zero torus area one obtains a type IIB in ten dimensions. On the other hand if

one starts with twelve dimensions and compactifies on T^2, then $SL(2, Z)$ gets interpreted as the symmetry of the torus in ten dimensions [14]. It has been conjectured that a D-string (defined with a world-sheet gauge field) has a critical dimension twelve with metric signature $(2, 10)$ and is related to ten dimensional theory by null reduction [40].

In this paper, we consider a hypothetical dynamical theory in twelve dimensions by lifting a three-form gauge field and a four-form one from an eleven dimensional M-theory and a ten dimensional type IIB theory respectively. Our analysis is in the spirit of F-theory as an underlying theory for type IIB superstring. We study the D-branes in type IIB theory and find their gravitational counterpart in a specific F-theory. In this frame-work, the three-brane and the six-brane (electric-magnetic dual of two-brane) in F-theory can be identified with the M-(two)brane and its dual M-(five)brane respectively in eleven dimensions. However, the three-brane does not seem to be directly identified with the two-brane in F-theory. Thus, with a motivation for an unified picture of three-brane in F-theory, we need to propose for the existence of a spatial isometry in one of the transverse directions. As a consequence the six-brane may be redefined and a D-(five)brane in type IIB theory can be viewed as the wrappings of a seven-brane in F-theory on a torus. This is consistent with the observation that with torus compactification of the twelve dimensional theory, the modulus of the type IIB string theory in ten dimensions can be identified with the modular parameter of the torus. We find that there exist p-branes ($p = 1, 2, \ldots, 7$) with singularities and they can be shown to be related to three-brane by duality symmetries in twelve dimensions itself. Thus, our analysis suggests that D-branes (D-strings, self-dual three-brane and D-(five)brane) in type IIB string theory may be viewed as the wrappings of a three-brane on a torus in F-theory.

We organize the paper as follows. In section 2, we discuss about the construction of a $D = 12$ classical theory in the spirit of ref. [17] and write down the dynamics of the background fields [38]. We show that the $D = 12$ theory when compactified on a torus T^2 gives rise to a ten dimensional theory similar to type IIB string. We begin section 3, with a D-string of type IIB theory and explicitly show that it can be viewed as the wrappings of three-brane on a torus T^2 in F-theory. We find charged two-brane solution to the low-energy limit of F-theory and identify that with the three-brane solution by proposing an isometry in one of the transverse directions. In section 4, we present a self-dual three-brane to type IIB string theory and view that as the wrappings of five-brane on a torus in F-theory. In section 5, we study a D-(five)brane of type IIB string theory and identify with the six-brane in F-theory by proposing an isometry in one of the transverse directions. Finally in section 6, we summarize our results and conclude with a note that the D-branes in type IIB superstring theory may be viewed as the wrappings of three-brane on a torus in F-theory.

II. TWELVE DIMENSIONS

It is know that the $SL(2,Z)$ invariance in the field equations of type IIB in $D = 10$ provides hints about the existence of twelve dimensional theory. However there is no supergravity theory in $D = 12$. Inspite of the fact, it may be possible to discuss a hypothetical twelve dimensional theory (may be with some kind of fermionic symmetries) with additional constraints on the background fields. Recently supersymmetry in $D = 12$ has also been discussed [41]. With the present understanding, it is believed that the $D = 12$ theory should contain type IIB fields in $D = 10$ along with those of M-theory in $D = 11$. It is observed that the four-form potential (say D_4) of type IIB supergravity (zero-slope limit of type IIB superstring) and the four-form field strength (say $\tilde{F}_4$) of $D = 11$ supergravity (low-energy limit of M-theory) are lifted to $D = 12$ dimensions in order to accommodate both type IIB three-brane and M-theory five-brane. As a result, it is natural to view three-brane and five-brane electric-magnetic duality in $D = 12$. Thus, one finds an electric M-(five)brane as a magnetic three-brane in F-theory and vice-versa.

In order to account for the possible fields and their dynamics in twelve dimensions, we start with the geometrical coupling [17] and write down the Chern-Simons term :

$$S_{CS} = \int_{\mathcal{M}_{12}} \hat{D}_4 \wedge \hat{F}_4 \wedge \hat{F}_4 = \int_{\mathcal{M}_{12}} \hat{C}_3 \wedge \hat{F}_4 \wedge \hat{F}_5 \quad ; \tag{1}$$

where $\hat{D}_4$ is the four-form potential ($\hat{F}_5 = d\hat{D}_4$) and $\hat{F}_4 = d\hat{C}_3$ is the four-form field strength defined with a three-form potential $\hat{C}_3$ in $D = 12$. Henceforth we use *hat* on the fields to denote them in $D = 12$.

In fact there can be one more Chern-Simons coupling which is topological in nature by construction and is given by

$$T_{CS} = \int_{\mathcal{M}_{12}} \hat{F}_4 \wedge \hat{F}_4 \wedge \hat{F}_4 \quad . \tag{2}$$

With an assumption $\mathcal{M}_{11} = \partial\mathcal{M}_{12}$, the topological coupling (2) can be identified with that of M-theory. However the geometrical coupling (1) may be identified with that of M-theory and type IIB theory after compactifications on a circle S^1 and a torus T^2 respectively without such assumption. It is noted that the geometrical coupling in eq.(1) induces currents ${j^*}_3 = \hat{F}_4 \wedge \hat{F}_5$ and ${j^*}_4 = (1/2)\hat{F}_4 \wedge \hat{F}_4$ for the gauge fields $\hat{C}_3$ and $\hat{D}_4$ respectively. In presence of five and six brane sources (say $\hat{q}_5$ and $\hat{q}_6$), these currents are not conserved. However, redefining the currents appropriately it is shown [17] to be conserved.

To begin with the dynamics for the $D = 12$ theory [38], we streamline our discussions with the motivation for the F-theory as an underlying theory for type IIB string theory

[14]. We also utilize the fact that two of the two-form potentials in type IIB theory form a doublet under an $SL(2,Z)$ transformation. It is known that there can (only) be one-form potentials on a torus T^2 which originate from a three-form in $D=12$ and there is only one four-form potential in $D=10$. Thus, it is natural to expect two two-form potentials forming an $SL(2,Z)$ doublet in $D=10$ from the dimensional reduction of the $D=12$ three-form potential $\hat{C}_3$ on T^2. The four-form potential $\hat{D}_4$ is assumed to be that of $D=10$ and with necessary constraints assumed to describe a self-dual three-brane. As a consequence, one arrives at the Chern-Simons coupling in type IIB theory from that of $D=12$ in eq.(1) on T^2. In principle there may be other higher form of potentials in $D=12$. However for our purpose, we consider the truncated theory describing three-branes and five-branes. Also, the action should contain a scalar curvature $R^{(12)}$ and in this case we define it with a metric of signature $(2,10)$ [14]. Following the discussions, it is plausible to write down the action describing the hypothetical theory in $D=12$ [38] as

$$S_{12D} \equiv \int_{\mathcal{M}_{12}} d^{12}x\,\sqrt{-\hat{G}}\Big(R^{(12)} - \frac{1}{2.4!}\hat{F}_4^2 - \frac{1}{2.5!}\hat{F}_5\Big) + \int_{\mathcal{M}_{12}} \hat{D}_4\wedge\hat{F}_4\wedge\hat{F}_4 + \dots\ . \tag{3}$$

Let us consider a compactification on T^2 with signature $(1,1)$, so that the $D=10$ metric has the Minkowski signature $(1,9)$. We restrict the background configuration in $D=12$ by defining the fields in terms of that in $D=10$ as :

the metric $\hat{G}$ with signature $(2,10)$ [14]

$$\hat{G} = \begin{pmatrix} G & 0 \\ 0 & M \end{pmatrix}\ ; \tag{4}$$

where G represents the metric with $(1,9)$ signature in $D=10$ and M with $(1,1)$ is the 2×2 matrix parameterizing the moduli fields in an $SL(2,Z)$ invariant form on a torus T^2. In our notation, the expression for the moduli field M in terms of scalars (the axion χ and the dilaton Φ) is given by

$$M = \begin{pmatrix} e^{\Phi} & \chi e^{\Phi} \\ \chi e^{\phi} & \chi^2 e^{\Phi} + e^{-\Phi} \end{pmatrix}\ ; \tag{5}$$

where the modulus $\lambda = \chi + ie^{-\Phi}$ and det $M=1$. We write the three-form potential

$$\hat{C} = \begin{pmatrix} 0 & B^q \\ B^p & 0 \end{pmatrix} \tag{6}$$

and the four-form potential as

$$\hat{D} = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}\ . \tag{7}$$

Where B^p, B^q for $p = 1, 2$ and D are the two-form potentials and a four-form one in ten dimensions respectively.

Now, we write the effective action in ten dimensions by compactifying the one in twelve dimensions (3) on a torus T^2 as :

$$S_{10D} \equiv \int_{\mathcal{M}_{10}} d^{10}x\sqrt{-G}\Big[R^{(10)} + \frac{1}{4}Tr\Big(\partial_\alpha M \partial^\alpha M^{-1}\Big) \\ - \frac{1}{2.3!}H^{(p)}M_{pq}H^{(q)} - \frac{1}{2.5!}{F_5}^2\Big] + \int_{\mathcal{M}_{10}} D_4 \wedge dB^1 \wedge dB^2 \quad ; \qquad (8)$$

where the metric G is in Einstein frame defined with $(1, 9)$ signature and $H^p = dB^p$ for $p = 1, 2$ are the $SL(2, Z)$ invariant field strengths corresponding to the two-form potentials. The scalar curvature $R^{(10)}$ and the moduli field M consisting of two scalars (identified with the axion χ and the dilaton Φ in eq.(5)) are also $SL(2, Z)$ invariant forms and obtained directly from the curvature $R^{(12)}$ using the compactifications in eq.(4). The five-form field strength in eq.(8) is not a self-dual one as required by the type IIB superstring theory. However, modulo the five-form field strength F_5 the action in eq.(8) may be identified with that of type IIB theory without the self-dual field strength in $D = 10$. Since, there is no covariant form of the type IIB action in $D = 10$, we identify the equations of motion corresponding to eq.(8) with the type IIB equations [39] by assuming necessary constraints on the background fields in $D = 12$. In principle the self duality of the five-form field strength F_5 and the supersymmetry in $D = 10$ have to come naturally from $D = 12$. These issues need deeper understanding of the subject and remain unanswered in the present frame-work of $D = 12$. However the analysis of classical solutions in $D = 12$ representing the gravitational counter part of the well established D-branes of type IIB string is nevertheless interesting. We analyze the wrappings of charged branes on a torus in F-theory and hope that our results may shed some light in understanding the conjectured F-theory.

In a different context, it is shown [17] that the three and four-form potentials are not independent in twelve dimensions in order to obtain a type IIB self-dual field strength. It is also plausible to consider (only) three-brane and its electric-magnetic dual five-brane in the frame-work of twelve dimensional theory of gravity due to natural reason. However in this frame-work, we find two-brane solutions corresponding to a three-form potential in addition to the three-brane corresponding to a four-form one in F-theory. In fact, we find that a three-brane in F-theory can be identified with a M-(two)brane in $D = 11$ which is dual to a M-(five)brane. Furthermore, the M-(five)brane can be lifted to $D = 12$ and may be viewed as a six-brane which is dual to the two-brane in F-theory. Thus following a via route through $D = 11$, it is possible to argue for an unified picture describing a three-brane in F-theory. On the other hand in the frame-work of $D = 12$ itself, we show that the two-brane

of F-theory can be identified with the three-brane by proposing an isometry in one of the transverse directions.

In order to find connections between the classical solutions to eq.(3) in $D = 12$ and that of eq.(8) in $D = 10$, we write the invariant distance in $D = 12$ as :

$$d\hat{s}^2(\mathcal{M}_{12}) = ds^2(\mathcal{M}_{10}) + ds^2(T^2) \tag{9}$$

where $\mathcal{M}_{12}$ is the Minkowski space of signature $(2, 10)$ [14] in $D = 12$, $\mathcal{M}_{10}$ is the one with the standard signature $(1, 9)$ in $D = 10$ and the $2nd$ term represents the line element on T^2 with the metric $(1, 1)$ defined in eq.(5). In our discussions of classical solutions, we consider the metric in $D = 12$ with $(2, 10)$ signature. However a physical signature $(1, 11)$ can be obtained by Wick rotating one of the time coordinates.

In a recent work [38], it is shown that a D-instanton of type IIB appears as a gravitational wave in $D = 12$. A D-instanton in type IIB theory can be described by the RR scalar (axion χ) which has its origin in the scalar curvature $R^{(12)}$ in $D = 12$. We present the classical solution in $D = 12$ corresponding to a D instanton in our notations as :

$$d\hat{s}^2 = -\Big(1 - \frac{Q_{-1}}{\mathbf{r}^8}\Big)dt^2 + \Big(1 + \frac{Q_{-1}}{\mathbf{r}^8}\Big)dy^2 - \frac{2Q_{-1}}{\mathbf{r}^8}dydt + d\mathbf{r}^2 + \mathbf{r}^2 d\Omega_9{}^2 \ \ ; \tag{10}$$

where t is the time-like coordinate and y is the spatial coordinate defined on a torus. The radial coordinate $\mathbf{r}$ is defined over the ten of the transverse coordinates. The global electric charge Q_{-1} is the D-instanton charge and can be interpreted as a linear momentum in $D = 12$. The signature of the metric is $(1, 11)$ which may be obtained by rotating one of time coordinates $(t \rightarrow iy)$ in a metric with $(2, 10)$ signature. Upon compactification on a torus with signature $(1, 1)$, the $D = 10$ describes an euclidean space $(0, 10)$.

Thus a D-instanton of type IIB theory has a gravitational counterpart in $D = 12$. The case of 7-branes of type IIB theory has already been addressed in ref. [14]. In the subsequent sections, we analyze and discuss about the $D = 12$ gravitational counterparts of D-string, its electric-magnetic dual D-(five)brane and the self-dual three-brane of type IIB string theory.

III. D-STRING AS A THREE-BRANE

In this section, we begin with a D-string solution [26,29] to the type IIB equations motion in $D = 10$ [39]. We set the RR scalar χ, the self-dual five-form field strength F_5 and the three-form field strength H in the NS-NS sector as well as all the fermionic fields to zero. Then we write down the the extremal D-string configuration consisting of the metric $(1, 9)$, the two-form potential B_2 in the RR sector and the dilaton Φ as :

$$
\begin{aligned}
ds^2 &= H_1(\mathbf{r})^{-\frac{3}{4}}\Big[-dt^2+dy^2\Big]+H_1(\mathbf{r})\Big[d\mathbf{r}^2+\mathbf{r}^2 d\Omega_7{}^2\Big] \ , \\
B_2 &= \pm H_1(\mathbf{r})^{-1} \ , \\
e^{\Phi} &= H_1(\mathbf{r})^{\frac{1}{2}} \\
\text{with} \qquad H_1(\mathbf{r}) &= 1+\frac{q_1}{\mathbf{r}^6} \ ; \qquad (11)
\end{aligned}
$$

where y-coordinate is parallel to string and defines the worldsheet along with the time-like coordinate t. The radial coordinate $\mathbf{r}$: $\mathbf{r}^2 = \delta_{ij}x^i x^j$ is defined with the orthogonal coordinates $(i,j=1,2,\ldots,8)$ to string and the angular part described by $d\Omega_7{}^2$ is the $SO(7)$ invariant line element on S^7. The global electric charge q_1 corresponds to the two-form potential B_2 in the RR sector. $H_1(\mathbf{r})$ is the harmonic function of transverse coordinates and the metric has a singularity at $\mathbf{r}=0$.

Now consider a $D=12$ theory in eq.(3) and we are interested to obtain a three-brane solution. It is natural to expect that the D-string in type IIB may appear as the wrappings of twelve dimensional three-brane on a torus T^2. We restrict the background configurations by setting the four-form field strength to zero ($\hat{F}_4=0$) and solve for the equations of motion in eq.(3). The consistent background configuration consists of the metric, the four-form potential $\hat{D}_4$ and can be given as :

$$
\begin{aligned}
d\hat{s}^2 &= \hat{H}_3(\mathbf{r})^{-\frac{1}{2}}\Big[-dt_a dt^a+dy_b dy^b\Big]+\hat{H}_3(\mathbf{r})^{\frac{1}{3}}\Big[d\mathbf{r}^2+\mathbf{r}^2 d\Omega_7{}^2\Big] \\
\hat{D}_4 &= \pm \hat{H}_3(\mathbf{r})^{-1} \\
\text{with} \qquad \hat{H}_3(\mathbf{r}) &= 1+\frac{\hat{q}_3}{\mathbf{r}^6} \ ; \qquad (12)
\end{aligned}
$$

where t_a ; $a=1,2$ are the time-like coordinates and y_b ; $b=3,4$ are the spatial coordinates defining the worldvolume of signature $(2,2)$ for the three-brane. The radial coordinate $\mathbf{r}$ is defined on the transverse plane over rest of the eight coordinates. The last term in eq.(12) describes the $SO(7)$ invariant line element on S^7. The conserved electric charge $\hat{q}_3$ corresponds to the four-form potential $\hat{D}_4$ in $D=12$. The harmonic function $\hat{H}_3(\mathbf{r})$ is defined with the transverse coordinates and can be shown to possess a point singularity at $\mathbf{r}=0$. The solutions in eq.(12) describes an electrically charged three-brane in $D=12$. By Wick rotating one of time-like coordinates and then compactifying on one of the spatial coordinates on the world-volume, the three-brane in F-theory may be identified with M-(two)brane in $D=11$. This is a consequence of the fact that the M-(two)brane corresponding to the four-form gauge field is lifted to $D=12$. In the asymptotic limit ($i.e.\mathbf{r}\rightarrow\infty$) the three-brane becomes flat.

Rescaling the D-string metric in eq.(11) with respect to the dilaton Φ

$$
G' = e^{-\frac{4\Phi}{3}}G \ , \qquad (13)
$$

we rewrite the metric as :

$$ds^2 = H_1(\mathbf{r})^{-\frac{2}{3}} H_1(\mathbf{r})^{-\frac{3}{4}} \Big[- dt^2 + dy^2 \Big] + H_1(\mathbf{r})^{\frac{1}{3}} \Big[d\mathbf{r}^2 + \mathbf{r}^2 d\Omega_7{}^2 \Big] \quad . \tag{14}$$

Since $SL(2,Z)$ invariance of the field equations of type IIB string gives rise to a theory in $D=12$, it should contain a $D=10$ metric and the other two dimensions arise from a torus with the modular parameter λ defining the moduli matrix M. Following the discussions in the previous section, we lift the D-string configurations to $D=12$ by writing down the metric as :

$$\begin{aligned} d\hat{s}^2 = H_1(\mathbf{r})^{-\frac{2}{3}} H_1(\mathbf{r})^{-\frac{3}{4}} \Big[- dt^2 + dy^2 \Big] - H_1(\mathbf{r})^{\frac{1}{2}} dz_1{}^2 \\ + H_1(\mathbf{r})^{-\frac{1}{2}} dz_2{}^2 + H_1(\mathbf{r})^{\frac{1}{3}} \Big[dr^2 + r^2 d\Omega_7{}^2 \Big] \end{aligned} \tag{15}$$

where z_1 is the time-like and z_2 is the spatial coordinate on the torus. Redefining the worldsheet coordinates (t,y) along with z_1 appropriately, we identify the background metric (15) with that of $D=12$ in eq.(12). In the process the two-form potential B_2 in eq.(11) is lifted to $D=12$ and is identified with the four-form potential $\hat{D}_4$ in eq.(12). Note that the harmonic function $H_1(\mathbf{r})$ in eq.(11) is identical to that of $\hat{H}_3(\mathbf{r})$ in eq.(12). The D-string electric charge q_1 gets interpreted as a three-brane charge $\hat{q}_3$ in $D=12$. Thus we obtain an electrically charged three-brane (12) with world-volume signature $(2,2)$ in $D=12$ starting from a D-string (11) in $D=10$. In other words, a three-brane in $D=12$ after double dimensional reduction on T^2 gives rise to a D-string of type IIB. Our analysis suggests that the D-string in type IIB theory may have its origin in F-theory and can be viewed as the wrappings of a charged three-brane on a torus. In addition, the three-brane in F-theory may also be viewed as a M-(two)brane in $D=11$. Then following an electric-magnetic duality in $D=11$, M-(two)brane can be identified with the M-(five)brane which may be lifted to $D=12$ to represent a six-brane and finally identified with the two-brane in F-theory. However in $D=12$ itself the two-brane does not seem to be related to three-brane in F-theory.

To understand the nature of two-brane in $D=12$, we consider the case with vanishing four-form potential $\hat{D}_4=0$ and with non-zero three form potential $\hat{C}_3$. We solve for the equations of motion (3) and obtain the background configuration consisting of the metric and the three-form potential $\hat{C}_3$ as :

$$\begin{aligned} & d\hat{s}^2 = \hat{H}_2(\mathbf{r})^{-\frac{2}{3}} \Big[- dt^a dt_a + dy^2 \Big] + \hat{H}_2(\mathbf{r})^{\frac{2}{7}} \Big[d\mathbf{r}^2 + \mathbf{r}^2 d\Omega_8{}^2 \Big] \ , \\ & \hat{C}_3 = \pm H_2(\mathbf{r})^{-1} \\ \text{with} \qquad & \hat{H}_2(\mathbf{r}) = 1 + \frac{\hat{q}_2}{\mathbf{r}^7} \end{aligned} \tag{16}$$

where $t_a \quad ; a = 1, 2$ are the time-like coordinates and y is the spatial coordinate defining the world-volume for the two-brane. The radial coordinate $\mathbf{r}$ is defined on the transverse plane with nine orthogonal coordinates and $d{\Omega_8}^2$ corresponds to the $SO(8)$ invariant line element on S^8. The conserved electric charge $\hat{q}_2$ corresponds to the three-form potential $\hat{C}_3$ in $D = 12$. The solution in eq.(16) describes an electrically charged two-brane in $D = 12$ with a point singularity at $\mathbf{r} = 0$ and is asymptotically flat.

In order to have an unified picture in $D = 12$, it is essential to relate the two-brane (16) with the three-brane in F-theory. As a consequence, D-string in type IIB theory can also be viewed as a two-brane in F-theory. With the motivation, we propose for the existence of an isometry in one of the transverse directions to that of two-brane in $D = 12$. Then the harmonic function $\hat{H}_2(\mathbf{r})$ can be reduced to a function of eight of the transverse coordinates instead of nine. The modified harmonic function can be written as

$$\hat{H}'_2(\mathbf{r}) = 1 + \frac{\hat{q}_2}{\mathbf{r}^6} \quad .$$

Then the metric in eq.(16) can be rewritten as :

$$d\hat{s}^2 = \hat{H}'_2(\mathbf{r})^{-\frac{2}{3}} \Big[- dt_a dt^a + dy^2 \Big] + dx^2 + \hat{H}'_2(\mathbf{r})^{\frac{2}{7}} \Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_7}^2 \Big] \quad . \tag{17}$$

The three-form potential retains its expression in eq.(16) with the modified harmonic function as defined above. The metric in eq.(17) along with the gauge field in eq.(16) describes a charged two-brane and can be shown to be identified with the three-brane in F-theory. The two-brane may also be related to M-(two)brane by a spatial compactification in x-direction.

In order to view the D-string in eq.(11) directly as a two-brane in F-theory, we rescale the D string metric as :

$$G'' = e^{-\frac{10\Phi}{7}} G \quad . \tag{18}$$

Following similar arguments as before, we lift the D-string solution in eq.(11) with the rescaled metric in eq.(18). Redefining the worldsheet coordinates appropriately, we identify the D-string backgrounds in $D = 12$ with that of two-brane (17) in F-theory. In this case, the electric charge of D-string can also be identified with that of two-brane charge in $D = 12$. Similarly, it is possible to identify the two-brane backgrounds directly with the three-brane in F-theory once an isometry is established in one of the orthogonal directions to the two-brane.

We summarize our results in this section with the observation that a D-string in type IIB theory can be viewed as the wrappings of three-brane on torus in F-theory. By proposing an isometry in one of the transverse directions, the two-brane in F-theory is shown to be

identified with the three-brane in $D = 12$. As a result, the three and four-form gauge fields describing the two and three-brane respectively in F-theory are not independent of each other. This is consistent with the observation [17] where it is argued to obtain a self-dual field strength in type IIB theory from $D = 12$. Our proposal for isometry leading to the identification of a two-brane with the three-brane in $D = 12$ itself suggests for an unified picture of three-brane in F-theory. It may be shown that a four-brane arises from a three-brane in F-theory due to the proposed isometry in one of the transverse directions to the three-brane world-volume.

IV. SELF-DUAL THREE-BRANE AS A FIVE-BRANE

In this section, we show that the self-dual three-brane in IIB string theory appears as a five-brane in $D = 12$ which is infact reduces to a type IIB theory on a torus T^2. Thus for the purpose, we consider a type IIB superstring and restrict to self-dual three-brane with vanishing one-form potentials along with the RR scalar and the fermionic sector. The background configuration consists of the metric, the self-dual four-form potential D_4 and the dilaton Φ. Solving the type IIB equations of motion [39], one obtains the extremal solution [29] as :

$$\begin{aligned} ds^2 &= H_3(\mathbf{r})^{-\frac{1}{2}}\Big[-dt^2 + dy_s dy^s\Big] + H_3(\mathbf{r})^{\frac{1}{2}}\Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_5}^2\Big] \ , \\ D_4 &= \pm H_3(\mathbf{r})^{-1} \ , \\ \Phi &= const. \\ \text{with} \qquad H_3(\mathbf{r}) &= 1 + \frac{q_3}{\mathbf{r}^4} \end{aligned} \tag{19}$$

where t is time-like and y_s ; $s = 1, 2, 3$ are the spatial coordinates defining a three-brane world-volume of signature $(1, 3)$. The radial coordinate $\mathbf{r}$ is defined with the transeverse coordinates x^i $(i, j = 1, 2, \ldots, 6)$ to D-(three)brane and the last term represents the $SO(5)$ invariant line element on S^5. The charge q_3 corresponds to the self-dual four-form potential $D_4 = {D^*}_4$ and thus carry both electric and magnetic charge simultaneously. The extremal backgrounds in eq.(19) describing a self-dual three-brane may be identified to that in ref. [42].

Let us consider the twelve dimensional theory (3) with non-zero five-form field strength and vanishing four-form field strength $\hat{F}_4 = 0$. We dualize the field strength and obtain the required seven-form field strength $\hat{F}^*_7$ to describe a five-brane in $D = 12$. We solve for the background fields containing the metric and six-form potential $\hat{D}^*_6$ (3) and obtain :

$$d\hat{s}^2 = \hat{H}_5(\mathbf{r})^{-\frac{1}{3}}\Big[-dt_a dt^a + dy_b dy^b\Big] + \hat{H}_5(\mathbf{r})^{\frac{1}{2}}\Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_5}^2\Big] \ ,$$

$$\hat{D}^*_6 = \pm \hat{H}_5(\mathbf{r})^{-1}$$

$$\text{and} \qquad \hat{H}_5(\mathbf{r}) = 1 + \frac{\hat{q}_5}{\mathbf{r}^4} \tag{20}$$

where t_a ; $a = 1, 2$ are the time-like and y_b ; $b = 3, 4, 5, 6$ are the spatial coordinates parallel to the five-brane defining the world-volume. The radial coordinate $\mathbf{r}$ is defined of the transverse coordinates $(i, j = 1, 2, \ldots, 6)$ orthogonal to the five-brane and the angular part is described by Ω_5 on S^5. The topological magnetic charge $\hat{q}_5$ corresponds to the six-form potential ${D^*}_6$ in (20). The solution in eq.(20) describes a magnetically charged five-brane in $D = 12$ and the metric can be shown to possess a point singularity at $\mathbf{r} = 0$. The five-brane solution in $D = 12$ is also asymptotically flat.

In order to identify the self-dual three-brane with the five-brane in $D = 12$ on T^2, we lift the $D = 10$ solution in eq.(19) to $D = 12$ by adding two extra dimensions on a torus following (9) which is in fact expressed in terms of moduli matrix (5) in $D = 10$. However in the case of self-dual three-brane, the moduli fields are constants. Thus, it is straightforward to lift the metric in eq.(19) to $D = 12$ and can be given by

$$d\hat{s}^2 = H_3(\mathbf{r})^{-\frac{1}{3}} \Big[- dt^2 + dy_s dy^s \Big] - d{z_1}^2 + d{z_2}^2 + H_3(\mathbf{r})^{\frac{1}{2}} \Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_5}^2 \Big] \ ; \tag{21}$$

where z_1 is the time-like and z_2 is the spatial coordinates defined on the torus. Redefining the the world-volume coordinates along with z_1 and z_2 appropriately, we arrive at the background metric in $D = 12$ as :

$$d\hat{s}^2 = H_3(\mathbf{r})^{-\frac{1}{3}} \Big[- dt_a dt^a + dy_s dy^s \Big] + H_3(\mathbf{r})^{\frac{1}{2}} \Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_5}^2 \Big] \ . \tag{22}$$

Comparing the harmonic function in eq.(19) with that of eq.(20), we notice that self-dual charge q_3 may be identified with the magnetic charge $\hat{q}_5$. Then the three-brane metric in eq.(22) along with the four-form potential and dilaton in eq.(19) may be identified with the five-brane solution in eq.(20). This analysis suggests that a self-dual three-brane may be viewed as a five-brane in a $D = 12$ theory and the self-dual three-brane charge plays the role of a five-brane magnetic charge. In other words, a self-dual three-brane with world-volume signature $(1, 3)$ may appear as the wrappings of five-brane $(2, 4)$ signature on a torus in $D = 12$. Using the electric-magnetic duality the self-dual three-brane may also be related to the electric three-brane in F-theory.

V. D-(FIVE)BRANE AS A SIX-BRANE

In this section, we analyze the D-(five)brane to type IIB string theory and show that it can be viewed as a six-brane in twelve dimensions by proposing an isometry in one of

the transverse dimensions. We consider the D-(five)brane solution [29] to type IIB string equations of motion [39] by setting the two-form potential to zero in the NS NS sector ($B = 0$) along with that of four-form potential ($D_4 = 0$) and the scalar ($\chi = 0$) in RR sector. Thus, the only non-vanishing background fields are the metric, the six-form potential ${B^*}_6$ (which is dual to the two-form one) and the dilaton Φ. The extremal D-(five)brane configurations describing the above backgrounds can be obtained by solving the type IIB equations of motion and can be given as :

$$\begin{aligned} ds^2 &= H_5(\mathbf{r})^{-\frac{1}{4}}\Big[-dt^2 + dy_s dy^s\Big] + H_5(\mathbf{r})^{\frac{3}{4}}\Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_3}^2\Big] \quad , \\ {B^*}_6 &= \pm H(\mathbf{r})^{-1} \quad , \\ e^{\Phi} &= H_5(\mathbf{r})^{-\frac{1}{2}} \\ \text{with} \qquad H_5(\mathbf{r}) &= 1 + \frac{q_5}{\mathbf{r}^2} \end{aligned} \tag{23}$$

where t is the time-like and y_s ; $s = 1, 2, \ldots, 5$ are the coordinates parallel to the five-brane defining the six-dimensional world-volume. The radial coordinate $\mathbf{r}$ is defined with the orthogonal coordinates x^i ($i, j = 1, 2, \ldots, 4$) to D-(five)brane and $d{\Omega_3}^2$ represents the invariant line element on S^3. The topological magnetic charge q_5 corresponds to the six-form potential ${B^*}_6$ in the RR sector.

Now consider the twelve dimensional theory (3) with vanishing five-form field strength $\hat{F}_5 = 0$ and non-zero four-form field strength. In this section, we are interested to find six-brane which is dual to two-brane in $D = 12$. Six-brane is described by a seven-form potential with an eight-form field strength ($\hat{F}^*_8$). We solve for the background fields (3) consisting of metric and the seven-form potential $\hat{C}^*_7$ and obtain

$$\begin{aligned} d\hat{s}^2 &= \hat{H}_6(\mathbf{r})^{-\frac{2}{7}}\Big[-dt_a dt^a + dy_b dy^b\Big] + \hat{H}_6(\mathbf{r})^{\frac{2}{3}}\Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_4}^2\Big] \quad , \\ \hat{C}^*_7 &= \pm \hat{H}_6(\mathbf{r})^{-1} \\ \text{with} \qquad \hat{H}_6(\mathbf{r}) &= 1 + \frac{\hat{q}_6}{\mathbf{r}^3} \end{aligned} \tag{24}$$

where t_a ; $a = 1, 2$ are the time-like and y_b ; $b = 1, 2, \ldots, 5$ are the spatial coordinates parallel to the six-brane defining the world-volume with $(2, 5)$ signature. As before the radial coordinate $\mathbf{r}$: $\mathbf{r}^2 = \delta_{ij} x^i x^j$ is defined on the rest of the transverse coordinates x^i ($i, j = 1, 2, \ldots, 5$) to six-brane. $\hat{q}_6$ is the topological magnetic charge corresponding to the seven-form potential $\hat{C}^*_7$. It can be shown that the magnetically charged six-brane metric has a point singularity at $\mathbf{r} = 0$ and is asymptotically flat. The six-brane solution in eq.(24) when compactified on one of the spatial coordinates of world-volume may be identified with that of M-(five)brane in $D = 11$. Further a M-(five)brane is dual to a M-(two)brane and following discussions in the section 3, the six-brane can be identified with a three-brane in

F-theory. In fact this is consistent with the observation [17] where it is argued that the five-brane of M-theory is lifted and may be identified with the three-brane of F-theory.

In order to view the D-(five)brane in eq.(23) as a six-brane in eq.(24) in $D = 12$ itself, we consider a spatial isometry in one of the transverse directions to the six-brane world-volume. As a result the harmonic function is modified and dependent on four of the transverse coordinates instead of five and can be given by

$$\hat{H}'_6(\mathbf{r}) = 1 + \frac{\hat{q}_6}{\mathbf{r}^2}.$$

The metric in eq.(24) can be rewritten separating the symmetry axis (say x) from the rest of the transverse directions and is given by

$$d\hat{s}^2 = \hat{H}'_6(\mathbf{r})^{-\frac{2}{7}}\Big[-dt^2 + dy_b dy^b\Big] + dx^2 + \hat{H}'_6(\mathbf{r})^{\frac{2}{3}}\Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_3}^2\Big] \quad . \tag{25}$$

Redefining the world-volume coordinates along with the spatial coordinate x, the six-brane may be viewed as a seven-brane in $D = 12$. In order to identify the D-(five)brane in eq.(23) with the six-brane backgrounds consisting of the potential (24) and the metric (25), we rescale the D-brane metric as :

$$G' = e^{\frac{\Phi}{6}} G \quad . \tag{26}$$

Following eq.(9) we lift the D-brane metric and rewrite in $D = 12$ as :

$$\begin{aligned} d\hat{s}^2 = H_5(\mathbf{r})^{-\frac{1}{3}}\Big[-dt_a dt^a + dy_s dy^s\Big] - H_5(\mathbf{r})^{-\frac{1}{2}} d{z_1}^2 \\ + H_5(\mathbf{r})^{\frac{1}{2}} d{z_2}^2 + \hat{H}_5(\mathbf{r})^{\frac{2}{3}}\Big[d\mathbf{r}^2 + \mathbf{r}^2 d{\Omega_3}^2\Big] \end{aligned} \tag{27}$$

where z_1 is time-like and z_2 is the spatial coordinate on the torus. Redefining the world-volume coordinates along with z_1 and z_2, we identify the metric in eq.(27) along with the expression for gauge field in eq.(23) with that of eqs.(25) and (24) respectively. Thus a D-(five)brane with world-volume signature $(1,5)$ appears as a six-brane $(2,5)$ in $D = 12$ with an isometry in one of the transverse directions. The topological magnetic charge of D-(five)brane can be identified with that of six-brane.

We summarize this section with the observation that a D-(five)brane may be viewed as a six-brane in $D = 12$ by proposing an isometry in one of the transverse directions to the six-brane world-volume. In other words, a D-(five)brane appears as the wrappings of seven-brane on a torus in F-theory. There also exist one-brane solutions which are electric-magnetic dual to seven-brane in $D = 12$. The analysis suggests that D-(five)brane may be viewed as a three-brane in F-theory using the proposal for isometry along with the duality symmetries.

VI. DISCUSSIONS

In this paper, we have considered a twelve dimensional theory of gravity with four-form and five-form field strengths as an underlying theory for the type IIB superstring. The compactification of the twelve dimensional theory on a torus is presented and the modular parameter of the torus is identified with the modulus describing a complex scalar field in ten dimensions. It is argued that the resulting equations of motion in ten dimensions may be identified with those of type IIB superstring by constraining the background fields appropriately in twelve dimensions.

By analyzing a D-string solution to type IIB theory in $D = 10$ along with a three-brane solution in $D = 12$, it is shown that D-string appears as a three-brane in F-theory and the corresponding electric charges are identified. Thus the wrappings of three-brane in $D = 12$ around the internal torus gives rise to a D-string of type IIB theory. It is argued that the three-brane in F-theory can be identified with a M-(two)brane in $D = 11$ by compactifying one of the coordinates on the world-volume in this frame-work. Two-brane solutions are also obtained in $D = 12$ by solving the metric equation with the three-form gauge field. It is noticed that the two-brane and three-brane in F-theory do not seem to be identified with each other. However proposing the existence of an isometry in one of the transverse directions to the two-brane world-volume, it is shown to be identified with a three-brane in F-theory. Thus the proposal for an isometry allows one to have an unified picture of three-brane in $D = 12$ in this frame-work. Our analysis is also consistent with the observation [17] that the three and four-form gauge fields in $D = 12$ are not independent of each other in order to have a self-dual four-form gauge field in $D = 10$.

In this frame-work, it is also shown that a self-dual three-brane can be viewed as a five-brane in $D = 12$. In the process of lifting a $D = 10$ theory to $D = 12$, the self-dual charge gets interpreted as a magnetic five-brane charge. Using the electric-magnetic duality, it is argued that a self-dual three-brane can also be viewed as an electrically charged three-brane in F-theory. We note that in the process of identifying the lifted self-dual three-brane with the five-brane, we do not need to rescale the three-brane metric (the dilaton is a constant) unlike the D-string and D-(five)brane cases. The identification of background fields in this case may allow one to think that the five-brane in F-theory is a direct consequence of the self-dual three-brane in Einstein frame.

Finally, we have shown that a D-(five)brane when lifted to $D = 12$ can be viewed as a six-brane by proposing the existence of an isometry in one of the transverse directions to six-brane world-volume. Then the magnetic charge of D-(five)brane can be interpreted as that of six-brane. At first sight this case seems to be in a different footing than the other

two cases where the D branes are viewed as the wrappings of higher branes on an internal torus in $D = 12$. However with the existence of an isometry, a six-brane can be related to a seven-brane in $D = 12$ and a similar picture for the D-(five)brane may be viewed. A six brane is dual to a two-brane in $D = 12$ and in turn may be identified with a three-brane in F-theory. By compactifying one of the time-coordinates of the six-brane world-volume, it can be shown to represent a M-(five)brane in $D = 11$. This is a consequence of the fact that the $D = 12$ theory of gravity is constructed by taking into account the M-(five)branes in $D = 11$ along with the self-dual three-brane of type IIB theory. Thus the analysis suggests that a magnetically charged M-(five)brane appears as an electrically charged three-brane and a self-dual three-brane as a five-brane in F-theory which are indeed electric-magnetic duals in $D = 12$.

We have not succeeded in listing the constraints on the background metric and gauge fields in $D = 12$, so that one obtains the desired type IIB fields in $D = 10$ directly upon dimensional reduction on T^2. Among various other issues, the origin of supersymmetry in type IIB remain unanswered in this frame-work of twelve dimensional theory of gravity. Since the extremal solutions to the type IIB theory are viewed as the three-brane in the low-energy limit of F-theory, it may be interesting to find out the corresponding fermionic symmetries in $D = 12$. We note that with the existence of an isometry there are additional charged p-branes in $D = 12$. Nonetheless, it is possible to relate all the p-branes ($p = 1, 2 \ldots, 7$) in $D = 12$ itself due to the existence of an isometry. Our analysis suggests that the D-branes of type IIB string theory may be reformulated and in turn can be viewed as a three-brane in F-theory.

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